

Breakdown of Fermi Liquid Theory in Doped Mott Insulators by Dynamical Spectral Weight Transfer

Philip Phillips, Ting-Pong Choy, and Robert G. Leigh

Department of Physics, University of Illinois 1110 W. Green Street, Urbana IL 61801, U.S.A.

We show that doped Mott insulators exhibit a collective degree of freedom, not made out of the elemental excitations, because the number of single-particle addition states at low energy per electron per spin is greater than one. The presence of such a collective degree of freedom which is not a consequence of proximity to a phase transition but rather stems from dynamical spectral weight transfer from high to low energies. This physics is captured by the charge $2e$ boson that emerges by explicitly integrating out the high-energy scale in the Hubbard model. The charge $2e$ boson binds to a hole, thereby mediating new charge e states at low energy. It is the presence of such charge e states which have no counterpart in the non-interacting system that provides the general mechanism for the breakdown of Fermi liquid theory in doped Mott insulators. The relationship between the charge $2e$ boson formulation and the standard perturbative treatment is explained.

The cornerstone of the standard theory of metals, Fermi liquid theory, is that although the electrons interact, the low-energy excitation spectrum stands in a one-to-one correspondence with that of a non-interacting system. In the normal state of the copper-oxide high-temperature superconductors, drastic deviations from the Fermi liquid picture obtain. The most dramatic effects include the 1) presence of a pseudogap^{1,2,3} in the single-particle spectrum, 2) broad spectral features in the electron-removal spectrum⁴, and 3) the ubiquitous T -linear resistivity⁶ above the superconducting transition. In the electronic state that accounts for this physics, the key assumption of Fermi liquid theory that the low-energy spectra of the interacting and free systems bare a one-to-one correspondence must break down. More precisely, the interacting system must contain electronic states at low energy that cannot be derived from the non-interacting system. As Polchinski⁷ and others⁸ have shown breaking Fermi liquid theory is notoriously difficult. Explicitly, aside from Cooper pairing, all 4-fermion interactions are irrelevant⁷ once one posits that electrons are the charge carriers. Hence, some fundamentally new degree of freedom is needed. How might such new degrees of freedom arise? One possibility¹² is if spectral weight transfer between high and low energies mediates new electronic states at low energy. Such states will have no counterpart in the non-interacting system, thereby leading to a breakdown of Fermi liquid theory. In this paper, we show explicitly that this state of affairs obtains quite generally in a doped Mott insulator. The new electronic state at low energies correspond to a bound state between a doubly occupied site and a hole. This new electronic state is mediated by the collective charge $2e$ boson which emerges¹³ in the exact low-energy theory of a doped Mott insulator described by the Hubbard model. Our work stands in contrast to a recent continuum non-Fermi liquid proposal^{14,15} whose relationship to lattice Mott physics is not established.

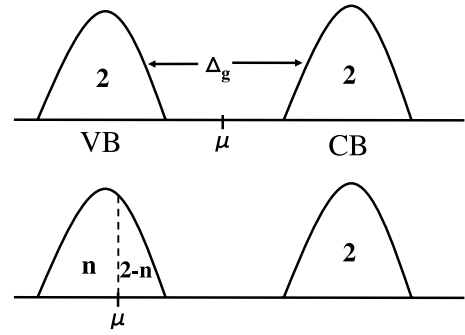


FIG. 1: Evolution of the single-particle density of states in a Fermi liquid in the valence band as a function of the electron filling, n . The total weight of the valence band is a constant 2, that is, 2 states per site. Doping simply pushes one state above the chemical potential. The integral of the density of states below the chemical potential is always the filling, n .

I. STATE COUNTING IN FERMIL LIQUIDS

In a non-interacting system, the number of low-energy addition states per electron per spin is equal to one. Should the number of low-energy addition states per electron per spin exceed unity, Fermi liquid theory fails and new electronic states emerge at low energy that cannot be constructed from the non-interacting system. To show that this state of affairs obtains in a doped Mott insulator, we compare the number of electrons per site (n_h) that can be added to the holes created by the dopants with the number of single-particle addition states per site at low energy,

$$L = \int_{\mu}^{\Lambda} N(\omega) d\omega, \quad (1)$$

defined as the integral of the single-particle density of states ($N(\omega)$) from the chemical potential, μ , to a cut-

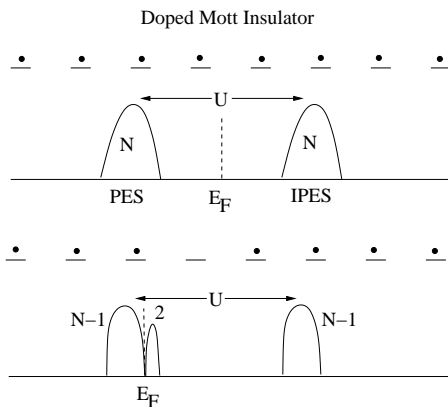


FIG. 2: Evolution of the single-particle density of states from half-filling to the one-hole limit in a doped Mott insulator described by the Hubbard model. Removal of an electron results in two empty states at low energy as opposed to one in the band-insulator limit. The key difference with the Fermi liquid is that the total weight spectral weight carried by the lower Hubbard band (analogue of the valence band in a Fermi liquid) is not a constant but a function of the filling.

off energy scale, Λ , demarcating the IR and UV scales. Consider first the case of a Fermi liquid or non-interacting system. As illustrated in Fig. (1), the total weight of the valence band is 2, that is, there are 2 states per site. The integrated weight of the valence band up to the chemical potential determines the filling. Consequently, the unoccupied part of the spectrum, which determines L , is given by $L = 2 - n$. The number of electrons that can be added to the empty sites is also $n_h = 2 - n$ (see Fig. (2)). Consequently, the number of low-energy states per electron per spin is identically unity. The key fact on which this result hinges is that the total weight of the valence band is a constant independent of the electron density.

II. DOPED MOTT INSULATORS: NOT JUST ELECTRONS

For a doped Mott insulator, the situation is quite different as a charge gap splits the spectrum into lower and upper Hubbard bands (LHB and UHB, hereafter) depicted in Fig. (2). At half-filling the chemical potential lies in the gap. The sum rule that 2 states exist per site applies only to the combined weight of both bands. At any finite doping, the weight in the LHB and UHB is determined by the density. For example, at half-filling each carries half the spectral weight. Even in the atomic limit, the spectral weights in the LHB and UHB are density dependent as shown in Fig. (2). Nonetheless, for a doped Mott insulator in the atomic limit, i.e. one electron per site with infinite on-site repulsion U , it is still true that $L/n_h = 1$, because creating a hole leaves behind an empty site which can be occupied by either a spin-up or a spin-down electron. Hence, when x electrons (see Fig. (3)) are removed, $L = n_h = 2x = 2 - 2n$ in the

atomic limit. Recall for a Mott system, $x = 1 - n$ as the hole doping occurs relative to half-filling. Hence, for a Mott system in the atomic limit, the total weight in the LHB increases from $1/2$ at half-filling to $1 - x + 2x = 2 - n$ as the system is doped. This result illustrates that the total weight in the LHB goes over smoothly to the non-interacting limit when $n = 0$. That is, 2 states exist per site at low energy entirely in the LHB. The change from half the spectral weight at $n = 1$ to all the spectral weight residing in the LHB at $n = 0$ is a consequence of spectral weight transfer. The atomic limit, however, only captures the static (state counting) part of the spectral weight. In fact the $2x$ sum rule, in which $L/n_h = 1$, is captured by the widely used^{9,10,11} $t - J$ model of a doped Mott insulator in which no doubly occupied sites are allowed. However, real Mott systems are not in the atomic limit. Finite hopping with matrix element t creates double occupancy, and as a result empty sites with weight t/U . Such empty sites with fractional weight contribute to L . Consequently, when $0 < t/U \ll 1$, L is strictly larger than $2x$. The actual behaviour, red curve in Fig. (3), has been confirmed both experimentally by oxygen 1s x-ray-absorption spectroscopy¹⁶ and theoretically by exact diagonalization¹⁷ and perturbation theory¹⁸. The number of *electrons*¹⁹ that can be added at *low energy* remains twice the number of dopants, $n_h = 2x$, even when the hopping is finite. That is, the quantum fluctuations mediated by the hopping do not change the number of electrons that can be added at low energy. As a result, for a doped Mott insulator, $L/n_h > 1$. Consequently, in contrast to a Fermi liquid, simply counting the number of electrons that can be added does *not* exhaust the available phase space to add an electron at low energy. Thus, additional degrees of freedom at low-energy, not made out of the elemental excitations, must exist. They arise from the hybridization with the doubly occupied sector and hence must emerge at low energy from a collective charge $2e$ excitation. The new charge e state that emerges at low energy must correspond to a bound state of the collective charge $2e$ excitation and the hole that is left behind. It is the physics of this new charge e state that mediates the non-Fermi liquid behaviour in a doped Mott insulator.

In a series¹³ of recent papers, we found the collective charge mode in a doped Mott insulator by integrating out exactly the high-energy scale in the Hubbard model, thereby obtaining an exact description of the IR physics. Consistent with the physical argument presented above, the collective mode is a charge $2e$ bosonic field which mediates new electron dynamics at low energy and is not made out of the elemental excitations in the UV. In an attempt to clarify this theory, we establish the relationship between the standard perturbative approach and the physics mediated by the charge $2e$ boson. By comparing how the operators transform in both theories, we are able to unambiguously associate the charge $2e$ boson with dynamical (hopping-dependent) spectral weight transfer across the Mott gap^{17,20,21}. Further, the spectral

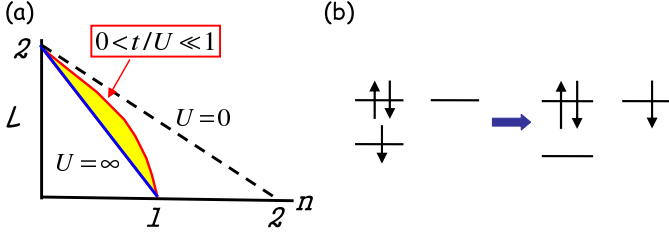


FIG. 3: a) Integrated low-energy spectral weight, L , defined in Eq. (3), as a function of the electron filling, n : 1) the dashed line is the non-interacting limit, vanishing on-site interaction ($U = 0$), in which $L = 2 - n$, 2) atomic limit (blue line) of a doped Mott insulator, $U = \infty$, in which $L = 2(1 - n) = 2x$, x the doping level and 3) a real Mott insulator in which $0 < t/U \ll 1$, red curve. For $0 < t/U \ll 1$, L must lie strictly above the $U = \infty$ limit and hence $L > 2x$ away from the atomic limit. (b) Hopping processes mediated by the t/U terms in the expansion of the projected transformed operators in terms of the bare electron operators (see Eq. (8)). As a result of the t/U terms in Eq. (9), the low-energy theory in terms of the bare fermions does not preserve double occupancy. The process shown here illustrates that mixing between the high and low-energy scales obtains only if double occupancy neighbours a hole. In the exact low-energy theory, such processes are mediated by the new degree of freedom, φ_i , the charge $2e$ bosonic field which binds a hole and produces a new charge e excitation, the collective excitation in a doped Mott insulator.

weight transfer is mediated by a bound excitation of the charge $2e$ boson and a hole in accord with the physical argument presented above.

III. STANDARD APPROACH

Of course the standard approach for treating the fact that $L/n_h > 1$ in a doped Mott insulator is through perturbation theory, not by explicitly constructing the missing degree of freedom. The goal of the perturbative approach is to bring the Hubbard model

$$H = -t \sum_{i,j,\sigma} g_{ij} a_{i,\sigma}^\dagger a_{j,\sigma} + U \sum_i a_{i,\uparrow}^\dagger a_{i,\downarrow}^\dagger a_{i,\downarrow} a_{i,\uparrow} \quad (2)$$

into block diagonal form in which each block has a fixed number of ‘fictive’ doubly occupied sites. Here i, j label lattice sites, g_{ij} is equal to one iff i, j are nearest neighbours and $a_{i,\sigma}$ annihilates an electron with spin σ on lattice site i . We say ‘fictive’ because the operators which make double occupancy a conserved quantity are not the physical electrons but rather a transformed (dressed) fermion we call $c_{i\sigma}$ defined below. Following Eskes et al.²⁰, for any operator O , we define \tilde{O} such that $O \equiv \mathbf{O}(a)$ and $\tilde{O} \equiv \mathbf{O}(c)$, simply by replacing the Fermi operators $a_{i\sigma}$ with the transformed fermions $c_{i\sigma}$. Note that O and \tilde{O} are only equivalent in the $U = \infty$ limit. The procedure which makes the Hubbard model block diagonal is now well known^{20,22,23,24,25}. One constructs

a similarity transformation S which connects sectors that differ by at most one ‘fictive’ doubly occupied site such that

$$H = e^S \tilde{H} e^{-S} \quad (3)$$

becomes block diagonal, where \tilde{H} is expressed in terms of the transformed fermions. In the new basis, $[H, \tilde{V}] = 0$, implying that double occupation of the transformed fermions is a good quantum number, and all of the eigenstates can be indexed as such. This does not mean that $[H, V] = 0$. If it were, there would have been no reason to do the similarity transformation in the first place. \tilde{V} , and not V , is conserved. Assuming that V is the conserved quantity results in a spurious local $SU(2)^{26,27}$ symmetry in the strong-coupling limit at half-filling.

Our focus is on the relationship between the physical and ‘fictive’ fermions. To leading order²⁰ in t/U , the bare fermions,

$$a_{i\sigma} = e^S c_{i\sigma} e^{-S} \simeq c_{i\sigma} - \frac{t}{U} \sum_{\langle j,i \rangle} [(\tilde{n}_{j\bar{\sigma}} - \tilde{n}_{i\bar{\sigma}}) c_{j\sigma} - c_{j\bar{\sigma}}^\dagger c_{i\sigma} c_{i\bar{\sigma}} + c_{i\bar{\sigma}}^\dagger c_{i\sigma} c_{j\bar{\sigma}}], \quad (4)$$

are linear combinations of multiparticle states in the transformed basis as is expected in degenerate perturbation theory. By inverting this relationship, we find that to leading order, the transformed operator is simply,

$$c_{i\sigma} \simeq a_{i\sigma} + \frac{t}{U} \sum_j g_{ij} X_{ij\sigma} \quad (5)$$

where

$$X_{ij\sigma} = \left[(n_{j\bar{\sigma}} - n_{i\bar{\sigma}}) a_{j\sigma} - a_{j\bar{\sigma}}^\dagger a_{i\sigma} a_{i\bar{\sigma}} + a_{i\bar{\sigma}}^\dagger a_{i\sigma} a_{j\bar{\sigma}} \right]. \quad (6)$$

What we would like to know is what do the transformed fermions look like in the lowest energy sector. We accomplish this by computing the projected operator

$$(1 - \tilde{n}_{i\bar{\sigma}}) c_{i\sigma} \simeq (1 - n_{i\bar{\sigma}}) a_{i\sigma} + \frac{t}{U} \sum_j g_{ij} [(1 - n_{i\bar{\sigma}}) X_{ij\sigma} - X_{ij\bar{\sigma}}^\dagger a_{i\bar{\sigma}} a_{i\sigma} - a_{i\bar{\sigma}}^\dagger X_{ij\bar{\sigma}} a_{i\sigma}]. \quad (7)$$

Simplifying, we find that

$$(1 - \tilde{n}_{i\bar{\sigma}}) c_{i\sigma} \simeq (1 - n_{i\bar{\sigma}}) a_{i\sigma} + \frac{t}{U} V_\sigma a_{i\bar{\sigma}}^\dagger b_i + \frac{t}{U} \sum_j g_{ij} [n_{j\bar{\sigma}} a_{j\sigma} + n_{i\bar{\sigma}} (1 - n_{j\bar{\sigma}}) a_{j\sigma} + (1 - n_{j\bar{\sigma}}) (a_{j\sigma}^\dagger a_{i\sigma} - a_{j\sigma} a_{i\sigma}^\dagger) a_{i\bar{\sigma}}]. \quad (8)$$

Here $V_\sigma = -V_{\bar{\sigma}} = 1$ and $b_i = \sum_{j\sigma} V_\sigma c_{i\sigma} c_{j\bar{\sigma}}$ where j is summed over the nearest neighbors of i . As is evident, the projected ‘fictive’ fermions involve the projected bare fermion, $(1 - n_{i\bar{\sigma}}) a_{i\sigma}$, which yields the $2x$

sum rule plus admixture with the doubly occupied sector mediated by the t/U corrections. These t/U terms, which are entirely local and hence cannot be treated at the mean-field level, generate the $> 2x$ or the dynamical part of the spectral weight transfer. This physics (which has been shown to play a significant role even at half-filling²⁸) is absent from projected models such as the standard implementation^{9,10,11} of the $t - J$ model in which double occupancy is prohibited. As we have pointed out in the introduction, the physics left out by projecting out double occupancy is important because it tells us immediately that $L/n_h > 1$; that is, new degrees of freedom must be present at low energy. Put another way, the operator in Eq. (8) is not a free excitation and but rather describes a non-Fermi liquid ($L/n_h > 1$). A process mediated by the last term in Eq. (8), depicted in Fig. (1), obtains only if a doubly occupied and empty site are neighbors. This underscores the fact that in Mott systems, holes can be heavily dressed by the upper Hubbard band. It is this dressing that generates dynamical spectral weight transfer.

Before we demonstrate how a single collective degree

of freedom describes such dressing, we focus on the low-energy Hamiltonian in the bare electron basis. The answer in the transformed basis is well-known²⁰ and involves the spin-exchange term as well as the three-site hopping term. Our interest is in what this model corresponds to in terms of the bare electron operators which do not preserve double occupancy. To accomplish this, we simply undo the similarity transformation after we have projected the transformed theory onto the lowest energy sector. Hence, the quantity of interest is $H_{sc} = e^{-S} P_0 e^S H e^{-S} P_0 e^S$. Of course, without projection, the answer in the original basis at each order of perturbation theory would simply be the Hubbard model. However, the question at hand is what does the low-energy theory look like in the original electron basis. Answering this question is independent of the high energy sectors in the transformed basis because all such subspaces lie at least U above the $m = 0$ sector. Hence, it is sufficient to focus on $P_0 e^S H e^{-S} P_0$. To express $P_0 e^S H e^{-S} P_0$ in the bare electron operators, we substitute Eq. (8) into the first of Eqs. (14) of Eskes, et al.²⁰ to obtain

$$\begin{aligned} H_{sc} &= e^{-S} P_0 e^S H e^{-S} P_0 e^S \\ &= -t \sum_{\langle i,j \rangle} \xi_{i\sigma}^\dagger \xi_{j\sigma} - \frac{t^2}{U} \sum_i b_i^{(\xi)\dagger} b_i^{(\xi)} - \frac{t^2}{U} \sum_{\langle i,j \rangle, \langle i,k \rangle, \sigma} \left\{ \xi_{k\sigma}^\dagger \left[(1 - n_{i\bar{\sigma}}) \eta_{j\sigma} + \xi_{j\bar{\sigma}}^\dagger \xi_{i\bar{\sigma}} \eta_{i\sigma} + \xi_{i\bar{\sigma}}^\dagger \xi_{i\sigma} \eta_{j\bar{\sigma}} \right] + h.c. \right\} \end{aligned} \quad (9)$$

as the low-energy theory in terms of the original electron operators. Here, $\xi_{i\sigma} = a_{i\sigma}(1 - n_{i\bar{\sigma}})$ and $\eta_{i\sigma} = a_{i\sigma} n_{i\bar{\sigma}}$. The first two terms correspond to the $t - J$ model in the bare electron basis plus 3-site hopping. However, terms in the bare-electron basis which do not preserve the number of doubly occupied sites explicitly appear. As expected, the matrix elements which connect sectors which differ by a single doubly occupied site are reduced from the bare hopping t to t^2/U . All such terms arise from the fact that the transformed and bare electron operators differ at finite U . Hence, Eq. (9) makes transparent that the standard implementation^{9,10,11} of the $t - J$ model in which the transformed and bare electron operators are assumed equal is inconsistent because the terms which are dropped are precisely of the same order, namely $O(t^2/U)$, as is the spin-exchange.

IV. NEW APPROACH

Explicitly integrating out¹³ the high-energy scale in the Hubbard model uncloaks the collective degree of freedom that accounts for the key difference between the bare and transformed electrons and the ultimate origin of the breakdown of Fermi liquid theory. The central element

of this theory is an elemental field, D_i , which we associated with the creation of double occupation via a constraint. Our approach is in the spirit of Bohm and Pines²⁹ who also extended the Hilbert space with a constrained field to decipher the collective behaviour of the interacting electron gas. A Lagrange multiplier φ_i , the charge $2e$ bosonic field, enters the action much the way the constraint σ does in the non-linear σ model. The corresponding Euclidean Lagrangian is

$$\begin{aligned} \mathcal{L} &= \int d^2\theta \left[\bar{\theta}\theta \sum_{i,\sigma} (1 - n_{i,-\sigma}) a_{i,\sigma}^\dagger \dot{a}_{i,\sigma} + \sum_i D_i^\dagger \dot{D}_i \right. \\ &\quad + U \sum_j D_j^\dagger D_j - t \sum_{i,j,\sigma} g_{ij} \left[C_{ij,\sigma} a_{i,\sigma}^\dagger a_{j,\sigma} + D_i^\dagger a_{j,\sigma}^\dagger a_{i,\sigma} D_j \right. \\ &\quad \left. \left. + (D_j^\dagger \theta a_{i,\sigma} V_\sigma a_{j,-\sigma} + h.c.) \right] + H_{\text{con}} \right] \end{aligned} \quad (10)$$

where $C_{ij,\sigma} \equiv \bar{\theta}\theta \alpha_{ij,\sigma} \equiv \bar{\theta}\theta (1 - n_{i,-\sigma})(1 - n_{j,-\sigma})$ and $d^2\theta$ represents a complex Grassman integration. The constraint Hamiltonian H_{con} is taken to be

$$H_{\text{con}} = s\bar{\theta} \sum_j \varphi_j^\dagger (D_j - \theta a_{j,\uparrow} a_{j,\downarrow}) + h.c. \quad (11)$$

The Grassman variable θ is needed to fermionize double occupancy so that it can properly be associated with the

high energy Fermi field, D_i . The constant s has been inserted to carry the units of energy. The precise value of s will be determined by comparing the low-energy transformed electron with that in Eq. (8). This Lagrangian was constructed so that if we solve the constraint, that is, integrate over φ and then D_i , we obtain exactly $\int d^2\theta\bar{\theta}\theta L_{\text{Hubb}} = L_{\text{Hubb}}$, the Lagrangian of the Hubbard model.

The advantage of this construction, however, is that we have been able to coarse-grain cleanly over the physics on the scale U . That is, all the physics on the scale U appears as the mass term of the new fermionic degree of freedom, D_i . It makes sense to integrate out D_i as it is a massive field in the new theory. The low-energy theory to $O(t^2/U)$,

$$\begin{aligned} H_{\text{eff}} = & -t \sum_{i,j,\sigma} g_{ij} \alpha_{ij\sigma} a_{i,\sigma}^\dagger a_{j,\sigma} \\ & - \frac{t^2}{U} \sum_j b_j^\dagger b_j - \frac{s^2}{U} \sum_i \varphi_i^\dagger \varphi_i \\ & - s \sum_j \varphi_j^\dagger a_{j,\uparrow} a_{j,\downarrow} - \frac{ts}{U} \sum_i \varphi_i^\dagger b_i + h.c. \quad (12) \end{aligned}$$

contains explicitly the charge $2e$ boson, φ_i . Here $b_i^{(a)} = \sum_{\sigma j} V_{\sigma} a_{i\sigma} a_{j\bar{\sigma}}$ where j is the nearest-neighbour of i . To fix the energy scale s , we determine how the electron operator transforms in the exact theory. As is standard, we add a source term to the starting Lagrangian which generates the canonical electron operator when the constraint is solved. For hole-doping, the appropriate transformation that yields the canonical electron operator in the UV is

$$\mathcal{L} \rightarrow \mathcal{L} + \sum_{i,\sigma} J_{i,\sigma} \left[\bar{\theta} \theta (1 - n_{i,-\sigma}) a_{i,\sigma}^\dagger + V_{\sigma} D_i^\dagger \theta a_{i,-\sigma} \right] + h.c.$$

However, in the IR in which we only integrate over the heavy degree of freedom, D_i , the electron creation operator becomes

$$\begin{aligned} a_{i,\sigma}^\dagger \rightarrow & (1 - n_{i,-\sigma}) a_{i,\sigma}^\dagger + V_{\sigma} \frac{t}{U} b_i a_{i,-\sigma} \\ & + V_{\sigma} \frac{s}{U} \varphi_i^\dagger a_{i,-\sigma} \quad (13) \end{aligned}$$

to linear order in t/U . This equation bares close resemblance to the transformed electron operator in Eq. (8), as it should. In fact, the first two terms are identical. The last term in Eq. (8) is associated with double occupancy. In Eq. (13), this role is played by φ_i . Demanding that Eqs. (8) and (13) agree requires that $s = t$, thereby eliminating any ambiguity associated with the constraint field. Consequently, the complicated interactions appearing in Eq. (9) as a result of the inequivalence between $c_{i\sigma}$ and $a_{i\sigma}$ are replaced by a single charge $2e$ bosonic field φ_i which generates dynamical spectral weight transfer across the Mott gap. The interaction in Fig. (1), corresponding to the second-order process in the term $\varphi_i^\dagger b_i$,

is the key physical process that enters the dynamics at low-energy. That the dynamical spectral weight transfer can be captured by a charge $2e$ bosonic degree of freedom is the key outcome of the exact integration of the high-energy scale. This bosonic field represents a collective excitation of the upper and lower Hubbard bands. However, we should not immediately conclude that φ_i gives rise to a propagating charge $2e$ bosonic mode, as it does not have canonical kinetics; at the earliest, this could be generated at order $O(t^3/U^2)$ in perturbation theory. Alternatively, we believe that φ appears as a bound degree of freedom. Since the dominant process mediated by φ_i requires a hole and a doubly occupied site to be neighbours (see Fig. (1)) we identify $\varphi_i^\dagger a_{i\bar{\sigma}}$ as a new charge e excitation responsible for dynamical spectral weight transfer. It is the appearance of this state at low-energy that accounts for the breakdown of Fermi liquid theory in a doped Mott insulator. Physically, φ_i is the dressing of a hole by the high-energy scale. We have previously shown¹³ that the formation of this bound state can produce the experimentally observed bifurcation of the electron dispersion below the chemical potential seen in PbBi2212³⁰, the mid-infrared band in the optical conductivity and the pseudogap³⁵. Further, the breakup of the bound state beyond a critical doping leads to T -linear resistivity³⁵.

The essential problem of Mottness is that in a hole-doped Mott insulator, empty sites can arise from doping or from hopping processes which mix the upper and lower Hubbard bands. Both contribute to L . However, the spectral weight on the empty sites resulting from mixing with the high-energy scale is proportional to t/U . Hence, such empty sites effectively represent holes with fractional charge $-e(t/U)$ not $-e$ as is the case with the holes resulting from doping. Consequently, they make no contribution to n_h , thereby giving rise to $L/n_h > 1$ for a doped Mott insulator and a general breakdown of the standard Fermi liquid theory of metals. At half-filling, such fractionally charged sites still persist. Adding an electron to such a system at low energies would require adding it coherently to a number of sites equal to $U/t \gg 1$. Such coherent addition of an electron at low energies has vanishing probability. The result is a gap for charge e but not for $e(t/U)$ excitations. At finite doping, holes in a Mott insulator are linear superpositions of both kinds of empty sites. As a result, holes in the hard-projected^{9,10,11} $t - J$ model, in which $L = 2x$, are not equivalent to holes in the Hubbard model. Approximations which prohibit explicit double occupancy, such as the standard treatment^{9,10,11} of the $t - J$ model in which the operators are not transformed, miss completely^{32,33,34} the localizing^{31,36,37} physics resulting from the orthogonality between charge e excitations and the sites with spectral weight t/U . In the exact theory, the physics associated with a finite length scale for double occupancy is contained straightforwardly in a charge $2e$ bosonic field, instead of being buried in complicated interaction terms in Eq. (9).

As Polchinski⁷ (as well as others⁸) have emphasized that from the point of view of the renormalization group, T -linear resistivity in the cuprates makes a Fermi liquid description untenable. We believe that our low energy theory containing the charge $2e$ bosonic field is in this sense a suitable replacement for Fermi liquid theory as it can explain³⁵ T -linear resistivity. We have shown above that the bosonic field accounts for what would be a consequence of complicated non-linear dependences on electron operators in projected models. What is clear from Polchinski's⁷ arguments is that projected models do not give a good basis upon which to build a theory – they mask the ubiquitous physics of strong coupling, namely

that new degrees of freedom emerge at low energy.

Acknowledgments

This work was initiated at the Kavli Institute for Theoretical Physics and funded partially through PHY05-51164. We thank George Sawatzky for several conversations that initiated this work, R. Bhatt, M. Hastings, R. Shankar, M. Sobol, A. Chernyshev, A. -M. Tremblay, and O. Tchernyshyov for helpful discussions and the NSF, Grant Nos. DMR-0605769.

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